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Macroscopic Quantum Coherent Phenomena in the Mesoscopic Electric Circuit

Bin Chen,[1,2] Xiaojuan Shen,[1] LiLy Sun,[1] and Rushan Han[3]

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The quantum theory for mesoscopic electric circuit with charge discreteness is briefly described. The Schrödinger equation of the mesoscopic electric circuit with external source which is the time function has been proposed. By using the instanton methods, the macroscopic quantum coherent phenomena and effective capacitance oscillation in the mesoscopic electric circuit have been addressed.

KEY WORDS: macroscopic quantum coherent phenomena; mesoscopic electric circuit.

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1. INTRODUCTION

Macroscopic quantum phenomena have been proposed for about twenty five years (Chudnovsky, 1993; Krive and Rozhavsky, 1992; Leggett, 1980). With the dramatic achievement in nano-technology, mesoscopic physics and nano-electronics are undergoing a rapid development (Büttiker, 1988; Landauer, 1988; Likharev, 1988). The electronic device community has been witnessing a strong and definite trend in the miniaturization of integrated circuits and components towards atomic-scale dimensions (Buot, 1993; Srivastava and Widom, 1987). When the transport dimension reaches a characteristic dimension, namely, the charge carrier inelastic coherence length, one must use quantum mechanics to discuss the problems in the mesoscopic systems and also need to consider the charge discreteness (Chen *et al.*, 1995; Louisell, 1973). In a previous paper, a quantum theory for mesoscopic electric circuits in accord with the discreteness of electric charges has been proposed (Li and Chen, 1996, 1998). Several years ago, M. Büttiker, Y. Imry and R. Landauer had predicted that the small and strictly one-

[1] Department of Physics, Hangzhou Teachers College, Hangzhou 310012, P. R. China.
[2] Department of Physics, Oklahoma State University, Stillwater, OK 74075, USA; e-mail: chenbin@hztc.edu.cn.
[3] Department of Physics, Peking University, Beijing 100873, P. R. China.

dimensional ring of normal metal, driven by an external magnetic flux, acts like superconductor rings with a Josephson junction, except that $2e$ is replaced by e (Büttiker *et al.*, 1983). Recent advances in micro-fabrication techniques have allowed the study of tunnel junctions with capacitance so low that the charging energy associated with a single electron can be several mev (Büttiker, 1988; Landauer, 1988; Likharev, 1988). Under appropriate condition, this charging energy can cause a suppression of tunnelling, called a Coulomb blockade, the other phenomena such as Bloch wave oscillation (Chen *et al.*, 1998) and some dynamic effects (Chen, Dai *et al.*, 2002; Chen, Juan *et al.*, 2003; Chen, Shen *et al.*, 1999). In present letter, we briefly demonstrate a quantum mechanical theory for mesoscopic electric circuits based on the discreteness of electric charge, and Josephson like effects in the mesoscopic circuit have also been shown, as well as the macroscopic quantum coherent phenomena and effective capacitance oscillation.

2. QUANTIZED ELECTRIC CIRCUIT WITH CHARGE DISCRETENESS

The classical equation of motion for an electric circuit of LC design is the same as that for a harmonic oscillation, whereas the "coordinate" means electric charge (Chen *et al.*, 1995; Louisell, 1973). The quantization of the circuit was carried out in the same way as that of a harmonic oscillator (Chen and Li *et al.*, 1995, 1996). In order to take into account the discreteness of electronic charge, we must impose that the eigenvalues of the self-adjoint operator $\hat{q}$ (electric charge) take discrete values,

$$\hat{q}|n\rangle = nq_e|n\rangle \tag{1}$$

where $n \in Z$ (set of integers) and q_e=1.602 $\times$ $10^{-19}c$, the elementary electric charge (Li and Chen, 1996, 1998). Since the spectrum of charge is discrete, the inner product in charge representation will be a sum instead of the usual integral and the electric current operator $\hat{P}$ will be defined by the discrete derivatives ∇_{q_e}, $\bar{\nabla}_{q_e}$. Thus for the mesoscopic quantum electric circuit one will have finite differential Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\Psi\rangle = \left[-\frac{\hbar^2}{2q_eL}(\nabla_{q_e} - \bar{\nabla}_{q_e}) + \frac{1}{2C}\hat{q}^2 + \epsilon(t)\hat{q}\right]|\Psi\rangle, \tag{2}$$

where L stands for inductance, C for the capacity and $\epsilon(t)$ for the voltage of an electric source which is always the time's function. The merits of this interdisciplinary approach are that the sinsoidal (Josephson effect) term appear naturally in the formalism which we will see below.

In order to solve the finite differential Schrödinger equation, the representation has been used

$$|\Psi\rangle = \sum_{n=-\infty}^{+\infty} C_n(t)|n\rangle. \tag{3}$$

Submitted (3) to (2), we get the equation

$$i\hbar\frac{\partial}{\partial t}C_n(t) = -\frac{\hbar^2}{2q_e^2 L}[C_{n+1}(t) + C_{n-1}(t)] + \frac{\hbar^2}{q_e^2 L}C_n(t) + \frac{n^2 q_e^2}{2C}C_n(t) + \epsilon(t) n q_e C_n(t). \tag{4}$$

In almost all experiments, it is the current that is under the best control. The current through a capacitor is given by

$$I = \frac{dQ}{dt}, \tag{5}$$

so that defining

$$a_n(t) = C_n(t)\exp\left\{\frac{i}{2\hbar}\int c\epsilon^2 dt\right\}. \tag{6}$$

Turning Eqs. (5) and (6) into (4), we get

$$i\hbar\frac{da_n}{dt} = \frac{(nq - Q)^2}{2C}a_n - \frac{\hbar^2}{2q_e L}(a_{n+1} + a_{n-1}) + \frac{\hbar^2}{q_e^2 L}a_n, \tag{7}$$

going into the pendulum representation

$$b(\theta, t) = \sum_{n=-\infty}^{+\infty} e^{in\theta}a_n(t), \tag{8}$$

now yields

$$i\hbar\frac{\partial b(\theta, t)}{\partial t} = \left\{\frac{[-iq_e\frac{\partial}{\partial\theta} - Q(t)]^2}{2C} + \frac{\hbar^2}{q_e^2 L}[1 - \cos\theta]\right\} b(\theta, t), \tag{9}$$

where $I(t) = dQ(t)/dt$ is the current biased. Note that the effective Hamiltonian for current biased mesoscopic electric circuit reads

$$H_{eff}(Q) = \frac{1}{2C}\left(-iq_e\frac{\partial}{\partial\theta} - Q\right)^2 + \frac{\hbar^2}{q_e^2 L}(1 - \cos\theta), \tag{10}$$

which is merely a time-varying canonical transformation of the effective Hamiltonian for a voltage electric circuit (Chen *et al.*, 1999). The duality between charge and flux now becomes evident. Solving the eigenvalue problem is present in Vourdas (1994), also present the Josephson like effect.

3. MACROSCOPIC QUANTUM COHERENT IN THE MESOSCOPIC ELECTRIC CIRCUIT

The quantum mechanical Lagrangian of the Hamiltonian for the mesoscopic electric circuit is

$$L = \frac{\hbar^2}{2q_e C}(\partial_t \theta)^2 - \frac{\hbar Q}{q_e C}\partial_t \theta - \frac{\hbar^2}{2q_e L}[1 - \cos\theta]. \tag{11}$$

The canonical momentum and the velocity

$$\pi_\theta = \frac{\hbar^2}{q_e^2 C}\dot{\theta} - \frac{\hbar Q}{q_e C}. \tag{12}$$

The trajectories at the circumference (S^{-1}) belong to the various homotopical nonequivalent classes that differ in the winding numbers n. Therefore the Euclidean functional for the statistical sum of the system under study contains an additional summation over the homotopic number is

$$Z = \sum_{n=-\infty}^{+\infty} \int [D\theta]_n \exp(-S_n), \tag{13}$$

where S_n is the Euclidean action in the given homotopic class, $n = \frac{1}{2\pi}\int_0^\beta d\tau\, \partial_t \theta$, where $\beta = \frac{1}{T}$ is the inverse temperature.

The $\theta(\tau)$ differing in the total winding number are physically equivalent. Therefore, the standard condition of periodicity (with the period β) of Bose-field with respect to the imaginary time in this case can be replaced by the quasi-periodic boundary condition

$$\theta_n(\tau + \beta) - \theta_n(\tau) = 2\pi n. \tag{14}$$

The equation of motion for the model (11) has exact instanton solution satisfying the boundary conditions (14)

$$\theta_n(\tau) = \pi + 2am\left(\frac{\omega_0 \tau}{k_n}\right), \tag{15}$$

where $am(z)$ is the elliptic amplitude and $\omega_0 = 2\frac{\hbar^2}{q_e^2}\sqrt{\frac{C}{L}}$. The modulus of the elliptic function k_n can be found from the relation

$$2nk_n K(k_n) = 2\frac{\hbar^2}{q_e^2}\sqrt{\frac{C}{L}}\beta, \tag{16}$$

where $K(k_n)$ is the complete elliptic 1st kind integral.The mechanical action of the model is

$$S_m = s_0\Phi(k_n) + in\left(-\frac{2\pi q_e Q}{c}\right), \tag{17}$$

where $s_0 = \frac{16}{\sqrt{LC}}$, $\Phi(k_n) = \frac{1}{k_n}[E(k_n) - \frac{1}{2}(1 - k_n^2)K(k_n)]$. Here S_0 is the single instanton action of the model and the $E(k)$ is an elliptic integral of the 2nd kind. The function $\Phi(k_n)$ depends on the homotopic index n only via the elliptic modulus $0 \leq k_n \leq 1$ and has the following values:

$$k_n \to 0, \quad \Phi(k_n) \simeq \frac{\pi}{4k_n}; \quad k_n^2 \to 1, \quad \Phi(k_n) \simeq 1 + \frac{1}{4}(1 - k_n^2). \tag{18}$$

The statistical *sum* in (13) and the free energy at low temperatures are further calculated in the standard approximation of dilute instanton gas (Rajaraman, 1982). The homotopic index is in this procedure merely the difference of the numbers of instantons (n_i) and auti-instantons ($\bar{n}_j$), $n = n_i - \bar{n}_j$. The final formula for the Q-depend contribution to the energy at $T \ll 2\frac{\hbar^2}{q_e^2}\sqrt{\frac{C}{L}}$ is as follows

$$\triangle E_Q = \frac{4\hbar^2}{q_e^2 L}\sqrt{\frac{2}{\pi}\sqrt{\frac{C}{L}}}e^{-\frac{16}{\sqrt{LC}}}\cos\frac{2\pi q_e Q}{C} \tag{19}$$

.

We have obtained the tunnel shift of the ground state energy (the contribution of instanton) which depend on the external voltage $\frac{Q}{C}$. This is macroscopic quantum coherent phenomena.

4. OSCILLATIONS OF THE EFFECTIVE CAPACITANCE IN THE MESOSCOPIC ELECTRIC CIRCUIT

The Hamiltonian of the mesoscopic electric circuit is Eq. (10). This Hamiltonian coincides with the quantum pendulum model with $\theta - vaccum$ and in terms of a fixed quasi-charge we have oscillatory effects similar to those considered for CDW's (Bogachek *et al.*, 1990). It is physically reasonable however to adjust the current in the circuit rather than the charge Q. In the case of low dc current this gives rise to the "Bloch oscillation" effect (Chen *et al.*, 1998).

We shall consider one more hypothetical type of mesoscopic oscillation in the electric circuit, biased by a fixed *dc* voltage. At a constant bias $V = \frac{Q}{C}$ across the circuit the phase θ is convenient to be represented as the sum of two terms: $\theta = \theta_0 + \chi$, i.e., the regular linearly increasing function of time, $\theta_0 = \frac{q_e}{\hbar} V t$, and the fluctuating term χ. Such representation is always possible for temperatures $T \ll \frac{\hbar^2}{2 q_e L}$. In this case rapid variations of the phase are associated with the regular classical part θ_0 and are induced by the external *dc* average voltage across the electric circuit, while the fluctuation term χ is a slow variable. At $T = 0$ the functional integral determining the vacuum-vacuum amplitude reduces to the integral over the fluctuations

$$Z = \int D\chi \exp\left[\frac{i}{\hbar} \int dt L(\theta = \theta_0 + \chi)\right], \tag{20}$$

and after averaging in the exponent (20) over the "rapid" time (with the period $\frac{\pi}{q_e V}$) determines the quantum dynamic of the slow subsystem (χ) described by the effective lagrangian:

$$L_{eff} = \frac{1}{2C} \frac{\hbar^2}{q_e^2} (\partial_t \chi)^2 + \frac{\hbar \epsilon}{q_e} (\partial_t \chi). \tag{21}$$

Therefore the asymptotic of the oscillating part of the free energy are given by the formula

$$\triangle F_\theta = 2T \exp\left(-\frac{\pi^2 T}{\epsilon_c}\right)\left[1 - cos\left(2\pi \frac{V}{V_c}\right)\right], \tag{22}$$

or

$$\triangle F_\theta = -\epsilon_c \left[\left[\frac{V}{V_c}\right]\right]^2, \tag{23}$$

where $[[x]]$is the fractional part of x to the nearest integer, $\epsilon_c = \frac{q_e^2}{2C}$ is the Coulomb energy due to vacuum fluctuation of the charge in capacitance. The second order derivative of the energy with respect to the voltage determines the effective capacitance of the electric circuit,

$$C_{eff}(V) = C\left[1 + 4\pi \frac{T}{\epsilon_c} \exp\left(-\pi^2 \frac{T}{\epsilon_c}\right) cos\left(2\pi \frac{V}{V_c}\right)\right], \tag{24}$$

which can be observed in high-frequency experiments (Lambe and Jaklev, 1969). Note that ϵ_c does not contain the plank constant, i. e. this effect of θ-vacuum represent a Coulomb effect.

5. CONCLUSION

Taking the electron discreteness into account, we studied the quantization of LC design mesoscopic electric circuit with external source which is the time function. The Schrödinger equation for LC design can become the well-known Mathieu equation, it can be exactly solved. The voltage biased and the current biased mesoscopic electric circuit have been discussed. Josephson like effects such as macroscopic quantum coherent phenomena and effective capacitance oscillation have been discussed in this mesoscopic electric circuit.

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